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Radion and moduli stabilization from induced brane actions in higher-dimensional brane worlds

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Abstract: We consider a $4+N$-dimensional brane world with 2 co-dimension 1 branes in an empty bulk. The two branes have $N-1$ of their extra dimensions compactified on a sphere $S^{(N-1)}$, whereas the ordinary 4 spacetime directions are Poincaré invariant. An essential input are induced stress-energy tensors on the branes providing different tensions for the spherical and flat part of the branes. The junction conditions — notably through their extra dimensional components — fix both the distance between the branes as well as the size of the sphere. As a result, we demonstrate, that there are no scalar Kaluza-Klein states at all (massless or massive), that would correspond to a radion or a modulus field of $S^{(N-1)}$. We also discuss the effect of induced Einstein terms on the branes and show that their coefficients are bounded from above, otherwise they lead to a graviton ghost.

Keywords: Extra Large Dimensions, Classical Theories of Gravity.

Contents

1. Introduction

The braneworld idea — where our 4 dimensional universe is part of a higher dimensional spacetime — is largely motivated from string theory and has attracted a lot of attention in recent years (see [1] for early work). In most proposed models standard matter particles are confined on a $d = 4$ dimensional braneworld whereas gravity propagates in the whole of spacetime.

Models with N compactified extra dimensions often suffer, however, from phenomenologically unacceptable massless scalars (moduli) associated to fluctuations of the size and the shape of the compactified manifold. For example, in a simple setup of a 5 dimensional spacetime with two 3-branes [2, 3] (and geometry $R^{(3,1)} \times S^1/Z_2$) a massless scalar is present, which is related to fluctuations of the distance between the branes [4, 5] and denoted the radion. Additional extra dimensions, parallel to the branes and compactified on a manifold $M^{(N-1)}$, lead then generically to additional massless moduli.

Such models with two co-dimension 1 branes are motivated by the strong coupling limit of the heterotic string [6, 7] and were formulated originally in $N = 7$ extra dimensions (with 6 among them along two 9-branes). Once one compactifies $N-1$ extra dimensions on $M^{(N-1)}$, the resulting geometry is thus $R^{(3,1)} \times M^{(N-1)} \times S^1/Z_2$. Even without including the additional degrees of freedom from tensor fields (or their scalar duals) in the bulk [8], one expects such models to suffer both from massless moduli and a massless radion.

Already long ago it was proposed to stabilize moduli through matter-induced quantum effects [9], and proposals to stabilize the radion range from classical matter in the bulk [10]–[12] to, again, Casimir-like forces involving matter in the bulk [13] or on the branes [14]. For recent proposals on stabilisation, based on bulk scalar fields, see [15].

In the present paper we consider pure gravity in a $4+N$-dimensional empty bulk with two co-dimension 1 branes. The branes consist of a 4 dimensional Poincaré invariant part and of $N-1$ extra dimensions compactified on a sphere (where $N > 2$). Thus the topology of the setup is $R^{(3,1)} \times S^{(N-1)} \times S^1/Z_2$.

The essential ingredient in our setup is the addition of terms on the brane actions originating from quantum effects of matter living on the branes. These terms will be sourcing the Einstein equations via the junction conditions warping the background spacetime. We consider in particular induced stress-energy tensors on the branes, that are constant and diagonal. At first sight these ressemble the "standard" brane tension (or cosmological constant). However, an overall cosmological constant or tension Λ corresponds to a stress-energy tensor $T_A^B = \delta_A^B \Lambda$, with indices A, B both in $R^{(3,1)}$ along the brane (denoted by $\{A,B\} = \{\mu,\nu\}$ subsequently) and in $S^{(N-1)}$ along the brane (denoted by $\{A,B\} = \{\alpha,\beta\}$ subsequently). The components of an induced stress-energy tensor [14] will, however, generically be different along $R^{(3,1)}$ or $S^{(N-1)}$, respectively,[1]

$$\begin{aligned} T^{(\text{ind})\ \nu}_{\quad\ \mu} &= \delta_\mu^{\ \nu}\, \Lambda^{(\text{ind},4)}\,, \\ T^{(\text{ind})\ \beta}_{\quad\ \alpha} &= \delta_\alpha^{\ \beta}\, \Lambda^{(\text{ind},S)}\,, \end{aligned} \tag{1.1}$$

with

$$\Lambda^{(\text{ind},4)} \neq \Lambda^{(\text{ind},S)}\,. \tag{1.2}$$

The origin of this difference lies in the fact, e.g., that expectation values of diagonal elements of operators like $\langle \partial_\mu \varphi \partial^\nu \varphi \rangle$ or $\langle \partial_\alpha \varphi \partial^\beta \varphi \rangle$ are generically different due to the different physical modes on $R^{(3,1)}$ or $S^{(N-1)}$ [14]. Alternatively, vevs of higher rank tensor fields could induce stress-energy tensors satisfying eq. (1.2) at the classical level. Due to (1.1) with (1.2) the induced stress-energy tensors cannot be absorbed completely by a redefinition of the brane tension Λ.

The aim of the present paper is to investigate, whether the inclusion of the above induced terms and the subsequent spacetime they induce, allows for a stabilised 3-brane universe. To this end we ask whether both the Einstein equations in the empty bulk and the junction conditions on the branes can be satisfied. We then study spin 2 fluctuations (following essentially the work of [23]) and study the scalar fluctuations of this system.

The essential results are as follows: first, one fine tuning condition among the tensions and stress-energy tensors of the branes has to be satisfied. This corresponds to the usual fine tuning of the effective 4d cosmological constant as for example in the Randall-Sundrum model. An essential difference here is, however, that only one fine tuning is required rather than two (one in each brane). Once this fine tuning is carried out, both the distance between the branes and the size of $S^{(N-1)}$ are fixed by the junction conditions. Hence, there are no massless modes associated to the radion or the overall modulus of $S^{(N-1)}$. In order for this mechanism to work the presence of induced stress-energy tensors is crucial; if they are switched off, the branes collapse and the 4 + N dimensional world volume vanishes.

[1] We are grateful to V. Rubakov for pointing this out.

Given the absence of massless scalars, it still could be that there are modes with negative mass^2, i.e. that the configuration is unstable. We show, however, that there are no 4d scalar modes at all — that are zero modes on $S^{(N-1)}$ and correspond to the radion and/or the modulus of $S^{(N-1)}$ — which could have potentially negative masses^2. Thus, the inclusion of induced stress-energy tensors satisfying (1.2) on the branes represents an economic and efficient mechanism in order to stabilize both the radion and moduli in higher dimensional brane worlds.

In addition we study the effect of induced Einstein terms as considered previously in [16, 17] (see also [18] for early work on induced gravity from quantum matter effects). Such curvature terms can be used to generate 4d gravity even in the case of one brane only embedded in an infinite volume flat bulk. They can lead to modification of gravity at very large scales [17, 19] which is a potentially interesting explanation to the dark energy component put forward by cosmological observations. This mechanism requires, however, a huge coefficient multiplying the induced Einstein term (compared to the fundamental gravitational scale in the bulk) which yields in turn a strong coupling problem [20] (see also [21] for recent developments). We will see, however, that in the present setup these coefficients have to be relatively small in order not to turn the 4d graviton into a ghost. Here 4d gravity will follow simply from the compactness of all extra dimensions.

The paper is organized as follows: in the next section we give the action, and the gravitational background that solves both the Einstein equations in the bulk (using previous results in [22, 23]) and the junction conditions on the branes (studied previously in [24]). We also study the "close brane limit", i.e. relatively small induced terms on the branes, leading to a relatively small distance between the branes or weak warping of spacetime. In this limit it is simpler to study the constraints on the parameters of the model. In section 3 we show that 4d gravity emerges and we give the expression for the 4d gravitational constant (the Planck scale) in terms of the fundamental parameters. Section 4 is devoted to a stability analysis of the setup, based on the check for the absence of tachyons (modes with negative mass^2), with the result announced above: there are no such modes at all. Details of the calculation (notably the wave equations for the gravitational fluctuations in the bulk, and their junction conditions) are referred to appendix A. In section 5 we study the effect of induced Einstein terms on the branes and show that they cannot play a dominant role. In section 6 we conclude with a summary and an outlook.

2. Higher-dimensional brane worlds with induced stress-energy tensors

We consider a $D = 4+N$ dimensional spacetime, with $N-1$ dimensions compactified on a sphere $S^{(N-1)}$. Along 4 dimensions spacetime is Poincaré invariant, whereas 1 dimension (perpendicular to the $D-2$-branes) is modded out by a Z_2 symmetry. The resulting topology is

$$R^{(3,1)} \times S^{(N-1)} \times S^1/Z_2\,. \tag{2.1}$$

Our conventions for the coordinates are as follows: x^μ, $\mu = 0, \ldots, 3$, are the 4 coordinates on $R^{(3,1)}$, z^α, $\alpha = 1, \ldots, N-1$, are the angles on $S^{(N-1)}$, y is the coordinate on S^1/Z_2.

Upper case latin letters run over all $D = 4 + N$ coordinates: $A, B, \ldots = \{\mu, \alpha, y\}$. We will consider 2 timelike branes of dimension $4 + (N - 1)$ situated at $y_1 = 0$ and $y_2 = \widehat{y}$, respectively. Coordinates along the branes are thus $\{x^\mu, z^\alpha\}$.

The bulk action includes the usual Einstein term $R^{(D)}$, but *no* cosmological constant. The actions on the branes 1 and 2 include the cosmological constants Λ_1 and Λ_2, and terms that give rise, in the Einstein equations, to induced stress-energy tensors satisfying eqs. (1.1) and (1.2). They follow from the coupling of gravity to composite operators of matter under the matter path integral. For simplicity we will not specify these operators, but denote them simply by $\Gamma_i^{(\mathrm{ind})}$ for $i = 1, 2$, which are defined by the generation of the induced stress-energy tensors $\Lambda_i^{(\mathrm{ind},4)}$ (along $R^{(3,1)}$), $\Lambda_i^{(\mathrm{ind},S)}$ (along $S^{(N-1)}$) in the Einstein equations below. Thus the action reads

$$S = \int d^4x\; d^{N-1}z dy \left\{ \sqrt{-g^{(D)}}\, \frac{1}{2\kappa_D}\, R^{(D)} + \sum_{i=1,2} \sqrt{-g^{(D-1)}}\, \delta(y - y_i) \left[\Lambda_i + \Gamma_i^{(\mathrm{ind})} \right] \right\}, \tag{2.2}$$

and the relevant components of the Einstein equations are

$$\frac{1}{\kappa_D} G_{\mu\nu}^{(D)} = \sum_{i=1,2} \sqrt{\frac{g^{(D-1)}}{g^{(D)}}}\, \delta(y - y_i) \left[g_{\mu\nu}\Lambda_i + g_{\mu\nu}\Lambda_i^{(\mathrm{ind},4)} \right], \tag{2.3a}$$

$$\frac{1}{\kappa_D} G_{\alpha\beta}^{(D)} = \sum_{i=1,2} \sqrt{\frac{g^{(D-1)}}{g^{(D)}}}\, \delta(y - y_i) \left[g_{\alpha\beta}\Lambda_i + g_{\alpha\beta}\Lambda_i^{(\mathrm{ind},S)} \right], \tag{2.3b}$$

$$\frac{1}{\kappa_D} G_{yy}^{(D)} = \frac{1}{\kappa_D} G_{\mu y}^{(D)} = \frac{1}{\kappa_D} G_{\alpha y}^{(D)} = 0 \tag{2.3c}$$

Let us first concentrate on the metric in the bulk, where the right-hand sides of eqs. (2.3) vanish. Given the spacetime symmetries of our configuration the metric reads,

$$ds^2 = A^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + B^2(y)d^2y + C^2(y)\gamma_{\alpha\beta}dz^\alpha dz^\beta \tag{2.4}$$

and $\gamma_{\alpha\beta}(z)$ is the metric on $S^{(N-1)}$ with radius 1. Solutions of the Einstein equations in the bulk for a metric of the form (2.4) have been obtained in [22]. Our form of the solutions is related to the ones in [22] (see also [23]) by a simple redefinition of the "radial" coordinate y, such that — in the present case — the action of the Z_2 symmetry becomes simply $y \to -y$. Defining a Z_2 invariant function $f(|y|)$,

$$f(|y|) = 1 + \alpha\, e^{-(N-2)\frac{|y|}{r_0}} \tag{2.5}$$

the solution reads,

$$\begin{aligned} A(y) &= f(|y|)^{n_A}\,, & n_A &= -\frac{1}{4}\sqrt{\frac{N-1}{N+2}}\,, \\ B(y) &= e^{\frac{|y|}{r_0}}\, f(|y|)^{n_B}\,, & n_B &= \frac{1}{N-2}\left(\sqrt{\frac{N-1}{N+2}} - \frac{N-3}{2}\right), \\ C(y) &= r_0\, e^{\frac{|y|}{r_0}}\, f(|y|)^{n_C}\,, & n_C &= \frac{1}{N-2}\left(\sqrt{\frac{N-1}{N+2}} + \frac{1}{2}\right). \end{aligned} \tag{2.6}$$

A few comments are now in order. Clearly the parameter α in eq. (2.5) can be gauged to 1 in the bulk, since one could eliminate it by a shift in y. However, the addition of brane 1 in the "gauge" $y_1 = 0$ breaks translational invariance in y, and then α has to be kept as a parameter.

A possible solution in a bulk without branes (before modding out by Z_2) is given by $f(y) = 1 + \alpha e^{-(N-2)\frac{y}{r_0}}$ and is therefore singular for $y \to -\infty$. We are cutting off this part of spacetime by introducing a brane at $y = y_1 = 0$. Accordingly parameter α dictates the portion of the bulk spacetime we cut off. A second parameter $r_0 > 0$ is related to the ADM energy of the background metric (2.4) much like a mass parameter in a black hole solution. It also characterises the size of the spheres, cf. the expression for $C(y)$ in (2.6). Finally, a third parameter consists in the position $y_2 = \widehat{y}$ of the second brane.

The solutions are well defined only for $N > 2$ since it is only then that the $(N-1)$-spherical subspace has nontrivial curvature. Finally the bulk metric would be asymptotically flat as $y \to \infty$. To put it in a nutshell the addition of the branes at $y_1 = 0$ and $y_2 = \widehat{y}$ cuts off the singular part of spacetime and asymptotic infinity respectively. Therefore the coordinate y varies over the finite interval $0 \le |y| \le \widehat{y}$ where the bulk metric is always non-singular. Finally Z_2 symmetry requires all functions $A(y)$, $B(y)$ and $C(y)$ to be even under $y \to -y$.

Next we have to consider the junction conditions, which follow from matching the singular terms in eqs. (2.3a) (along $R^{(3,1)}$) and (2.3b) (along $S^{(N-1)}$) for $y \to y_i$. At each brane one obtains two independent equations corresponding to eqs. (2.3a) and (2.3b), respectively. They read,

$$\frac{1}{\kappa_D B_i}\left(-3\left[\frac{A''}{A}\right]_i - (N-1)\left[\frac{C''}{C}\right]_i\right) = \Lambda_i + \Lambda_i^{(\mathrm{ind},4)}, \tag{2.7a}$$

$$\frac{1}{\kappa_D B_i}\left(-4\left[\frac{A''}{A}\right]_i - (N-2)\left[\frac{C''}{C}\right]_i\right) = \Lambda_i + \Lambda_i^{(\mathrm{ind},S)}, \tag{2.7b}$$

where

$$\left[\frac{A''}{A}\right]_i = \left(\frac{\partial_y A}{A}\bigg|_{y_i+\varepsilon} - \frac{\partial_y A}{A}\bigg|_{y_i-\varepsilon}\right) = 2\sigma_i \frac{A_i'}{A_i} \tag{2.8}$$

and similarly for $[C''/C]$. The last equality in (2.8) holds due to the Z_2 symmetry, and the sign σ_i is $\sigma_1 = +1$, $\sigma_2 = -1$. B_i denotes simply $B(y_i)$ etc. Using the expressions (2.6) for $A(y)$, $B(y)$ and $C(y)$, equations (2.7) can be rewritten as

$$\sqrt{f_i}(1-w) + \frac{1}{\sqrt{f_i}}(1+w) = \sigma_i \kappa_D C_i \left(\Lambda_i^{(\mathrm{ind},S)} - \Lambda_i^{(\mathrm{ind},4)}\right), \tag{2.9a}$$

$$\begin{aligned}\sqrt{f_i}\,(N-1+(N-3)w) + \frac{1}{\sqrt{f_i}}\,(N-1-(N-3)w) = \\ = -\sigma_i \kappa_D C_i \left(2\Lambda_i + (N-1)\Lambda_i^{(\mathrm{ind},S)} - (N-3)\Lambda_i^{(\mathrm{ind},4)}\right)\end{aligned} \tag{2.9b}$$

where $f_i = f(|y_i|)$, and

$$w = \frac{1}{2}\sqrt{(N-1)(N+2)}\,. \tag{2.10}$$

It is important to compare the number of equations to solve with the number of free parameters: equations (2.9) have to be satisfied at both branes 1 and 2, which gives 4 equations altogether. On the other hand the configuration has just 3 free parameters: r_0, α in the metric (in the function f in (2.5) and (2.6)) and the position $\widehat{y}$ of the second brane. Hence one fine tuning condition among the bare parameters Λ_i, $\Lambda_i^{(\mathrm{ind})}$, $i = 1,2$ in (2.9), is required for a solution to exist. This is nothing but the "usual" fine tuning of the cosmological constant. Here we have assumed that the gravitational background is Poincaré invariant along 3+1 dimensions; if the fine tuning condition was not satisfied, we would find 3+1 de-Sitter or anti-de-Sitter solutions.

Note, however, that here we have "just" *one* fine tuning to perform compared to *two* fine tunings that are required in the Randall-Sundrum model [2] with a cosmological constant in the bulk. The need to perform two fine tunings in this latter model can be traced back to the particular form of the logarithmic derivative of the warp factor in a portion of AdS space, which is *independent* of y. In this case the distance between the branes never enters the junction conditions, and consequently it is left free (corresponding to the massless radion), but two fine tunings are required. Here the junction conditions *fix* $\widehat{y}$, the distance between the branes. Hence there is no massless radion. Moreover, the radius of the compact space $S^{(N-1)}$ is fixed: according to the metric (2.3) it is given by $C(y)$ (and varies thus with y), but since all parameters as r_0 and α are determined by eqs. (2.9), $C(y)$ is completely fixed. Hence there is no massless modulus related to a fluctuating size of $S^{(N-1)}$.

The explicit expressions for the bulk parameters r_0, α and $\widehat{y}$ as functions of the bare parameters on the branes can be obtained from (2.9). Defining,

$$L_i \equiv 2\Lambda_i + (N-1)\Lambda_i^{(\mathrm{ind},S)} - (N-3)\Lambda_i^{(\mathrm{ind},4)}\,, \qquad \delta_i \equiv \frac{\Lambda_i^{(\mathrm{ind},S)} - \Lambda_i^{(\mathrm{ind},4)}}{L_i} \tag{2.11}$$

and

$$v = N - 1 + (N-3)w \tag{2.12}$$

(with w as in eq. (2.10)) we obtain

$$\alpha = \frac{2(1+(N-1)\delta_1)}{w - 1 - v\delta_1} \tag{2.13}$$

$$r_0 = -\frac{2(N-1) + v\alpha}{\kappa_D (1+\alpha)^{n_c+\frac{1}{2}} L_1} \tag{2.14}$$

with n_C as in eqs. (2.6). In addition it is useful to define

$$s_1 = \frac{1+(N-1)\delta_1}{1+(N-1)\delta_2}\,, \qquad s_2 = \frac{w-1-v\delta_1}{w-1-v\delta_2}\,, \qquad s_3 = \frac{1+w+(2(N-1)-v)\delta_1}{1+w+(2(N-1)-v)\delta_2}\,, \tag{2.15}$$

which gives

$$e^{(N-2)\frac{|\widehat{y}|}{r_0}} = \frac{s_1}{s_2} \tag{2.16}$$

and the fine tuning condition

$$\frac{L_1}{L_2} = -\frac{1}{s_2}\left(\frac{s_1}{s_2}\right)^{\frac{1}{N-2}}\left(\frac{s_2}{s_3}\right)^{n_c+\frac{1}{2}}\,. \tag{2.17}$$

Note that if the induced terms are equal ($\delta_i \to 0$ for $i = 1, 2$) eq. (2.15) leads to $s_1 = s_2 = s_3 = 1$, and thus eq. (2.16) implies $|\widehat{y}| = 0$. Hence, without the differing induced terms, the distance between the branes vanishes and the branes collapse. Non-vanishing induced terms — at least on one brane — are necessary for the present setup to exist.

Let us now consider the approximation $\Lambda_i^{\text{ind}}/\Lambda_i \ll 1$, i.e. relatively small induced stress-energy tensors, which simplifies the above equations. From eq. (2.11) this implies

$$|\delta_i| \ll 1 \tag{2.18}$$

both for $i = 1$ and 2. Then the junction conditions (2.9) give the following results to leading order in δ_i: the fine tuning condition (2.17) reads

$$\Lambda_2 = -\Lambda_1\,, \tag{2.19}$$

and for the parameters α, r_0 and $\widehat{y}$ one obtains (with w as in eq. (2.10))

$$\alpha = \frac{2}{w-1}\,, \tag{2.20a}$$

$$r_0 = -\frac{2w(N-2)}{\kappa_D \Lambda_1 (w+1)} \left(\frac{w-1}{w+1}\right)^{n_C - \frac{1}{2}}\,, \tag{2.20b}$$

$$|\widehat{y}| = \frac{2wr_0}{w-1}\,(\delta_1 - \delta_2)\,. \tag{2.20c}$$

In view of eq. (2.18) the distance between the branes is thus relatively small, and we will denote this situation as the "close brane limit" (to be studied again in section 5). Clearly, $r_0 > 0$ implies, from eqs. (2.20b) and (2.19), that $\Lambda_1 < 0$ and $\Lambda_2 > 0$, hence brane 1 is the negative tension brane. (This asymmetry originates from our convention of a negative exponent in the function f in eq. (2.5).) Here the leading order expressions for δ_i are

$$\delta_i \cong \frac{1}{2\Lambda_i}\left(\Lambda_i^{(\text{ind},S)} - \Lambda_i^{(\text{ind},4)}\right). \tag{2.21}$$

Hence the positivity of the right-hand side of eq. (2.20c) implies (with $\Lambda_1 < 0$, $\Lambda_2 > 0$)

$$\Lambda_1^{(\text{ind},4)} + \Lambda_2^{(\text{ind},4)} > \Lambda_1^{(\text{ind},S)} + \Lambda_2^{(\text{ind},S)}\,. \tag{2.22}$$

The inequality (2.22) is not difficult to fulfill, and is sufficient for the existence of a consistent solution of the junction conditions.

In the next section we consider the gravitational fluctuations around the background (2.4), and concentrate on the emergence of the 4d graviton.

3. Background fluctuations and 4d gravity

In order to obtain the linear equations for gravitational fluctuations, we expand the metric about the curved background $g_{AB}^{(0)}$ (2.4),

$$ds^2 = \left(g_{AB}^{(0)} + h_{AB}\right) dx^A dx^B\,, \tag{3.1}$$

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and use the Einstein equations (2.3). D-dimensional general coordinate invariance corresponds to the following gauge freedom:

$$h_{AB} \to h'_{AB} + \nabla_A \xi_B + \nabla_B \xi_A \tag{3.2}$$

where ∇_A are covariant derivatives computed with $g^{(0)}_{AB}$, and ξ_A are $D = 4 + N$ arbitary functions. To fix (partially) the gauge freedom (3.2) we choose the axial gauge,

$$h_{\mu y} = h_{\alpha y} = 0\,. \tag{3.3}$$

It is easily checked that the junction conditions do not involve the modes (3.3). Indeed the modes in (3.3) are odd under the Z_2-symmetry, and therefore vanish on both branes. In principle, the freedom (3.2) also allows to gauge h_{yy} to zero. However, there are also (non-gravitational) modes, that correspond to fluctuations of the brane positions around $y_1 = 0$, $y_2 = \widehat{y}$. We choose to use the remaining gauge degree of freedom to gauge these brane bending modes away, i.e. to keep the brane positions fixed. Then h_{yy} cannot be gauged away as well [4, 5].

With respect to a 4-dimensional observer bulk metric fluctuations can be seperated into scalar, spin 2 and "vector" like. Indeed the latter fluctuations with the index structure $h_{\mu\alpha}$ correspond to 4d vector fields. These will always contain massless states in 4d, which form a Yang-Mills sector with the gauge group $O(N-1)$, the isometrics of $S^{(N-1)}$. Subsequently we will not be interested in this sector. We are left with fluctuations with the index structures $h_{\mu\nu}$, h_{yy} and $h_{\alpha\beta}$, which generally depend on x^μ, y and z^α. Furthermore, we confine ourselves to the lowest mass states with respect to the angular mode excitations on $S^{(N-1)}$. The perturbations $h_{\mu\nu}$ and h_{yy} are then independent of the angles z^α on $S^{(N-1)}$, and $h_{\alpha\beta}$ is proportional to the metric $\gamma_{\alpha\beta}$ on $S^{(N-1)}$. It is convenient to rescale the linear fluctuations h_{AB} by the background metric, i.e. to define $\widehat{h}_{\mu\nu}$, $\widehat{h}_S$, $\widehat{h}_{yy}$ as

$$h_{\mu\nu}(x^\mu, y) = A^2(y)\,\widehat{h}_{\mu\nu}(x^\mu, y)\,. \tag{3.4}$$

$$h_{\alpha\beta}(x^\mu, y, z^\alpha) = \frac{C^2(y)}{N-1}\gamma_{\alpha\beta}(z_\alpha)\,\widehat{h}_S(x^\mu, y) \tag{3.5}$$

$$h_{yy}(x^\mu, y) = B^2(y)\,\widehat{h}_{yy}(x^\mu, y)\,. \tag{3.6}$$

The scalar modes $\widehat{h}_S$ and $\widehat{h}_{yy}$ represent fluctuations of the overall size of the sphere (the modulus) and the radion mode respectively.

The Einstein equations for the fluctuations can be split into two regimes: in the bulk, away from the branes, the vanishing of the D dimensional Einstein tensor implies the vanishing of all components of the Ricci tensor. In appendix A we give all components $R_{\mu\nu}$, R_{yy}, $\gamma^{\alpha\beta}R_{\alpha\beta}$ and $R_{\mu y}$ of the Ricci tensor linear in the fluctuations $\widehat{h}_{\mu\nu}$, $\widehat{h}_{yy}$ and $\widehat{h}_S$. $R_{\mu y}$ is related to the other components by a Bianchi identity, and $R_{\alpha y}$ vanishes identically for z^α-independent fluctuations. On the branes we have to consider the junction conditions for the fluctuations which are also given in appendix A.

Let us now turn to the massless spin 2 fluctuations with respect to a 4-d observer. Our approach follows closely that of [23], the essential difference here being the presence of two co-dimension 1 branes and of the induced curvature terms. These fluctuations contain a massless transverse tracefree mode $h_{\mu\nu}^{(grav)}$ representing the 4d graviton:

$$\partial^\mu h_{\mu\nu}^{(\mathrm{grav})} = \eta^{\mu\nu} h_{\mu\nu}^{(\mathrm{grav})} = 0\,. \tag{3.7}$$

The 4d graviton is easily identified [23] as the y independent mode of $\widehat{h}_{\mu\nu}$ satisfying (3.7). Neglecting the scalar fluctuations $\widehat{h}_\mu{}^\mu$, $\widehat{h}_{yy}$ and $\widehat{h}_S$, the wave equations (A.5b)–(A.5d) in appendix A are trivially satisfied. The wave equation (A.5a), originating from the vanishing of $R_{\mu\nu}$, reduces to

$$\Box^{(4)}\, \widehat{h}_{\mu\nu} = 0\,. \tag{3.8}$$

The junction conditions (A.7) and (A.8) are also satisfied, and this mode is trivially normalizable, since all extra dimensions are compact.

In order to obtain an expression for the 4d Planck mass we proceed as follows: first one defines a rescaled 4d metric $\widehat{g}_{\mu\nu}$ as

$$g_{\mu\nu} = A^2(y)\, \widehat{g}_{\mu\nu}\,. \tag{3.9}$$

Then we insert the expression (3.9) for the metric into the action (2.2), and integrate over all extra dimensions dy, $d^{N-1}z$. The result is an effective 4d action of the form

$$S_{\mathrm{eff}} = \int d^4x\, \sqrt{-\widehat{g}}\, \frac{1}{2\kappa_4} \widehat{R}^{(4)} \tag{3.10}$$

where $\widehat{R}^{(4)}$ is the Ricci scalar constructed from $\widehat{g}_{\mu\nu}$. Comparing the coefficients, one obtains for κ_4

$$\frac{1}{\kappa_4} = \Omega^{N-1} \frac{2}{\kappa_D} \int_0^{\widehat{y}} dy\, A^2(y)\, B(y)\, C(y)^{N-1} \tag{3.11}$$

where Ω^{N-1} is the volume of $S^{(N-1)}$ with radius 1, and the factor 2 in front of the integral over dy originates from the fact that originally it ranges from $-\widehat{y}$ to $+\widehat{y}$. This integral is always finite provided $\widehat{y}$ is finite and corresponds to the normalisation factor for the constant 4-dimensional graviton mode (and the subsequent massive spin 2 modes). The analysis of the massive spin 2 modes and higher angular momenta goes through as in [23] modulo the presence of the boundary terms. The spin 2 modes are stable provided the right hand side of eq. (3.11) is positive.

This is not quite the end of the story. Assuming that matter lives on one of the two branes 1 or 2, the original coupling of matter to gravity is of the form

$$\int d^4x\, d^{N-1}z \sqrt{-g^{(D-1)}}\, \mathcal{L}_{\mathrm{matter}}\left(g^{(D-1)}\right)\bigg|_i\,. \tag{3.12}$$

Assuming that the matter wave functions are zero modes on $S^{(N-1)}$, the $d^{N-1}z$ integral can be performed as before which turns (3.12) into

$$\Omega^{N-1}\, C_i^{N-1} \int d^4x \sqrt{-\widehat{g}}\, A_i^4\, \mathcal{L}_{\mathrm{matter}}\left(A_i^2 \widehat{g}_{\mu\nu}\right). \tag{3.13}$$

The powers of $A_i \equiv A(y_i)$ cancel exactly in (3.13) for scale invariant matter (kinetic terms of gauge fields), but have to be eliminated by a field redefinition in the case of non-scale invariant matter as scalars. As proposed in [2] this could generate mass scales that are naturally much smaller than the fundamental (gravitational) scales of the theory, if A_i is very small. We will not consider this possibility here. The prefactors $\Omega^{N-1}C_i^{N-1}$ in (3.13) can also be eliminated by a rescaling of the fields (such that their kinetic terms are properly normalized) and the couplings in the action. Recall that with our convention (2.4) for the metric, where z^α are dimensionless angles on $S^{(N-1)}$, $C(y)$ has the dimension of a length (cf. the last of eqs. (2.6)). Only after this last rescaling the fields and couplings assume their dimensions appropriate for a 4d effective theory.

4. Stability analysis of the scalar fluctuations

The absence of both a massless radion and a massless modulus associated to the size of $S^{(N-1)}$ follows from the fact that both the distance $\widehat{y}$ between the branes and the (y-dependent) radius $C(y)$ of $S^{(N-1)}$ are fixed, through the junction conditions, in terms of the parameters in the action. However, these (or other) scalar modes could acquire a negative (4d) mass^2, which would signal a tachyonic instability of the present configuration.

In order to check this, we have to study the lowest 4d scalar Kaluza-Klein states contained in the gravitational fluctuations $h_{\mu\nu}$, h_{yy} and $h_{\alpha\beta}$. As indicated in eqs. (3.4)–(3.8) in the previous section, we confine ourselves to zero modes on $S^{(N-1)}$. The crucial question is then whether there are modes $\varphi(x^\mu, y)$ (Kaluza-Klein states along the y direction) with possibly negative 4d mass^2. Our important result is that, somewhat astonishingly, there are no such modes at all (beyond pure gauge transformations), hence the present configuration is extremely rigid. This result is far from obvious, and will follow from a detailed analysis of the bulk equations of motion and the junction conditions.

First, the gravitational fluctuations $h_{\mu\nu}$, h_{yy} and $h_{\alpha\beta}$ do not represent the complete spectrum of potential physical fluctuations of the configuration: in addition there are brane bending modes $\beta_i(x^\mu)$, which correspond to fluctuations $y_i \to y_i + \beta_i(x^\mu)$ around the positions $y_i \equiv \{y_1, y_2\} = \{0, \widehat{y}\}$ of the branes (in a given coordinate system).

However, the gauge (3.3)

$$h_{\mu y} = h_{\alpha y} = 0 \tag{4.1}$$

still allows for general infinitesimal coordinate transformations

$$\delta x^\mu = \xi^\mu(x^\mu, y)\,, \qquad \delta y = \chi(x^\mu, y) \tag{4.2}$$

provided that

$$\eta_{\mu\nu}\,\frac{d\xi^\nu}{dy} = -\frac{B^2}{A^2}\,\frac{d\chi}{dx^\mu} \tag{4.3}$$

relating ξ^μ and χ so that the gauge condition (4.1) remains valid. This gauge freedom (4.2) can be partially used to gauge away the brane bending modes β_i, by a suitable choice of $\chi(x^\mu, y_i)$. In fact we are still left with general coordinate transformations of the form (4.2) with (4.3), as long as the gauge parameter $\chi(x^\mu, y)$ *vanishes on both branes*, i.e. satisfies

Dirichlet boundary conditions. This boundary gauge choice is used implicitely in the junction conditions (A.13)–(A.15) in the appendix, where the brane bending modes β_i have already been omitted.

It is convenient to decompose the gravitational fluctuations into plane waves along $R^{(3,1)}$, i.e.

$$h_{\mu\nu}, h_{yy}, h_{\alpha\beta} \sim e^{ik_\lambda x^\lambda} \tag{4.4}$$

where $k^2 = -m^2$, the mass of the mode in question (and one can assume $m^2 \neq 0$).

Out of these fluctuations we can construct four 4d scalars (and zero modes on $S^{(N-1)}$): $\widehat{h}_S$ and $\widehat{h}_{yy}$ have already been defined in eqs. (3.5) and (3.6), and out of $\widehat{h}_{\mu\nu} = A^{-2} h_{\mu\nu}$ we can project

$$\widehat{h}_4 = \eta^{\mu\nu}\, \widehat{h}_{\mu\nu} \tag{4.5}$$

and, for non-zero modes,

$$\widehat{L} = k^{-2} k^\mu k^\nu \widehat{h}_{\mu\nu} = -m^{-2} k^\mu k^\nu \widehat{h}_{\mu\nu} \tag{4.6}$$

(hence L, as defined in eq. (A.6), is related to $\widehat{L}$ through $L = m^2 A^{-2} \widehat{L}$).

The remaining general coordinate transformations (4.2) with (4.3) (and Dirichlet boundary conditions on χ) act on the four 4d scalars $\widehat{h}_4$, $\widehat{L}$, $\widehat{h}_{yy}$ and $\widehat{h}_S$ with masses m^2 as

$$\delta\widehat{h}_4(y) = 2m^2 \int_0^y dy' \, \frac{B^2}{A^2}\chi - 8\frac{A'}{A}\chi \,, \tag{4.7a}$$

$$\delta\widehat{L}(y) = 2m^2 \int_0^y dy' \, \frac{B^2}{A^2}\chi - 2\frac{A'}{A}\,\chi \,, \tag{4.7b}$$

$$\delta\widehat{h}_{yy}(y) = -2\left(\chi' + \frac{B'}{B}\chi\right) , \tag{4.7c}$$

$$\delta\widehat{h}_S(y) = -2(N-1)\frac{C'}{C}\chi \,. \tag{4.7d}$$

The bulk equations given in the appendix simplify somewhat when written in terms of the four modes above. First we define

$$a = A^4\, C^{N-1}\, B^{-1} \tag{4.8}$$

such that

$$D' \equiv 4\frac{A'}{A} + (N-1)\frac{C'}{C} - \frac{B'}{B} = a' a^{-1} \,, \tag{4.9}$$

and in addition we define

$$E' = 3\frac{A'}{A} + (N-1)\frac{C'}{C} \,. \tag{4.10}$$

Then, in an obvious notation, the bulk equations in the appendix can be rewritten as follows: $2aB^2\times$ (A.7) becomes

$$[a\widehat{h}_4']' + 4a\frac{A'}{A}\left(\widehat{h}_4' + \widehat{h}_S' - \widehat{h}_{yy}'\right) + a\frac{B^2}{A^2}m^2\left(2\widehat{h}_4 - 2\widehat{L} + \widehat{h}_S + \widehat{h}_{yy}\right) = 0 \,, \tag{4.11}$$

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$2aB^2\frac{A^2}{m^2}\times$ (A.8) becomes

$$[a\widehat{L}']' + a\frac{A'}{A}\left(\widehat{h}_4' + \widehat{h}_S' - \widehat{h}_{yy}'\right) + a\frac{B^2}{A^2}m^2\left(\widehat{h}_4 - \widehat{L} + \widehat{h}_S + \widehat{h}_{yy}\right) = 0\,, \tag{4.12}$$

$2aB^2\times$ (A.5c) becomes

$$\begin{aligned} &[a\widehat{h}_S']' + a(N-1)\frac{C'}{C}\left(\widehat{h}_4' + \widehat{h}_S' - \widehat{h}_{yy}'\right) + \\ &+a\frac{B^2}{A^2}m^2\widehat{h}_S + 2a\frac{B^2}{C^2}(N-2)\left(\widehat{h}_S - (N-1)\widehat{h}_{yy}\right) = 0\,, \end{aligned} \tag{4.13}$$

and $(-\frac{A^2}{m^2})\times$ (A.9) turns into

$$\widehat{L}' - \widehat{h}_4' - \widehat{h}_S' + \left(\frac{A'}{A} - \frac{C'}{C}\right)\widehat{h}_s + E'\widehat{h}_{yy} = 0\,. \tag{4.14}$$

There is no need to reconsider eq. (A.5b) as well, since the five equations (A.5b), (A.5c), (A.7)–(A.9) are related through the Bianchi identity. The above equations (4.11)–(4.13) are coupled wave equations for modes $\widehat{h}_4$, $\widehat{L}$, $\widehat{h}_S$ and $\widehat{h}_{yy}$. Out of the four independent equations (4.11)–(4.14) we can construct two more dependent equations, that prove to be useful subsequently: multiplying eq. (4.14) by a, taking the derivative with respect to y, using eqs. (4.11)–(4.13) and relations between the warp factors A, B and C given in eq. (2.6), one can derive another equation involving only first order derivatives with respect to y of the four modes:

$$\begin{aligned} &E'\widehat{h}_4' + \left(E' + \frac{A'}{A} - \frac{C'}{C}\right)\widehat{h}_S' + (N-2)\frac{B^2}{C^2}\widehat{h}_S - \\ &-(N-1)(N-2)\frac{B^2}{C^2}\widehat{h}_{yy} + \frac{B^2}{A^2}m^2\left(\widehat{h}_4 - \widehat{L} + \widehat{h}_S\right) = 0\,. \end{aligned} \tag{4.15}$$

Next, from $(N-1)\frac{C'}{C}\times$ (eq. (4.12)–eq. (4.11)) + $3\frac{A'}{A}\times$ eq. (4.13) and eliminating $\widehat{h}_{yy}$ using (4.14) one derives

$$\begin{aligned} &(N-1)\frac{C'}{C}\left[a(\widehat{L}' - \widehat{h}_4')\right]' + 3\frac{A'}{A}\left[a\widehat{h}_S'\right]' + \frac{B^2}{A^2}m^2\left[(N-1)\frac{C'}{C}a(\widehat{L} - \widehat{h}_4) + 3\frac{A'}{A}a\widehat{h}_S\right] + \\ &+6(N-2)\frac{A'B^2}{AC^2E'}\left[(N-1)a(\widehat{L}' - \widehat{h}_4' - \widehat{h}_S') + (N+2)\frac{A'}{A}a\widehat{h}_S\right] = 0 \end{aligned} \tag{4.16}$$

which will be used below.

It is straightforward to check that the above bulk equations are invariant under the gauge transformations (4.7). In order to evade this gauge ambiguity we now proceed as follows: first we construct three independent gauge invariant fields out of the four 4d scalars above. Then we study their bulk equations and solve for the two modes with respect to the third. Thus we show that the bulk equations can be reduced to a unique second order differential equation in y for one independent mode. Finally, we demonstrate that the junction conditions for this mode imply both Dirichlet and Neumann boundary conditions on both branes, hence there are no gauge invariant fluctuations at all.

Three independent gauge invariant fields H_1, H_2 and H_3 can be defined as follows:

$$H_1 = a\left[(N-1)\frac{C'}{C}(\widehat{L}-\widehat{h}_4)+3\frac{A'}{A}\widehat{h}_S\right] \tag{4.17a}$$

$$H_2 = a\left[\widehat{L}'-\widehat{h}_4'+\left(E'+\frac{A'}{A}\right)(\widehat{L}-\widehat{h}_4)+3\frac{A'}{A}\widehat{h}_{yy}\right] \tag{4.17b}$$

$$H_3 = a\left[3\frac{A'}{A}\widehat{h}_4'+\left(4\frac{A'}{A}\left(E'+\frac{A'}{A}\right)+m^2\frac{B^2}{A^2}\right)(\widehat{h}_4-\widehat{L})-12\frac{A'^2}{A^2}-\widehat{h}_{yy}\right]. \tag{4.17c}$$

The first y derivative of H_1 can be written as

$$H_1' = a\left[(N-1)\frac{C'}{C}(\widehat{L}'-\widehat{h}_4')+3\frac{A'}{A}\widehat{h}_S'+(N-1)(N-2)\frac{B^2}{C^2}(\widehat{L}-\widehat{h}_4)\right]. \tag{4.18}$$

Here, and in the following, we use relations between the warp factors A, B and C derived from eqs. (2.6). Now, the bulk equation (4.14) can be used to show that

$$H_1' = E'H_2+\left(\frac{A'}{A}-\frac{C'}{C}\right)H_1\,. \tag{4.19}$$

Next, from the bulk equation (4.15) one can derive

$$\left[\frac{(N+2)}{(N-1)}\frac{A'}{A}\left(E'+\frac{A'}{A}\right)+m^2\frac{B^2}{A^2}\right]H_1+3\frac{A'}{A}\left(E'+\frac{A'}{A}-\frac{C'}{C}\right)H_2-E'H_3=0\,. \tag{4.20}$$

From eqs. (4.19) and (4.20) one can obtain H_2 and H_3 in terms of H_1 and H_1', which is left as the only remaining independent degree of freedom. Finally, after some algebra the bulk equation (4.16) becomes an equation that contains H_1 only:

$$\begin{aligned} &E'H_1''+\left(E'D'-2(N-1)(N-2)\frac{B^2}{C^2}\right)H_1'+\\ &+\left(2(N+2)(N-2)\frac{B^2}{C^2}\frac{A'}{A}+\frac{B^2}{A^2}E'm^2\right)H_1=0\,. \end{aligned} \tag{4.21}$$

It remains to study the junction (boundary) conditions, subject to which eq. (4.21) has to be solved.

First, in terms of the above four 4d scalar non-zero modes the junction conditions (A.14)–(A.15) can be written as (here we neglect the effect of induced Einstein terms, that are discussed in the next section, and which are included for completeness in the junction conditions in the appendix)

$$\left[-(N-1)\widehat{h}_4'-(N-2)\widehat{h}_S'+(N-1)\left(E'+\frac{A'}{A}-\frac{C'}{C}\right)\widehat{h}_{yy}\right]_i=0\,, \tag{4.22a}$$

$$\left[-3\widehat{h}_4'-4\widehat{h}_S'+4E'\widehat{h}_{yy}\right]_i=0\,, \tag{4.22b}$$

$$\left[\widehat{L}'-\widehat{h}_4'-\widehat{h}_S'+E'\widehat{h}_{yy}\right]_i=0\,. \tag{4.22c}$$

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Using the bulk equation (4.14) in (4.22c), one derives

$$[\widehat{h}_S]_i = 0\,. \tag{4.23}$$

(Here one needs

$$\left[\frac{A'}{A} - \frac{C'}{C}\right]_i \neq 0\,,$$

which follows from the junction conditions (2.7).)

Next, from $\frac{A'}{A}\times$ eq. (4.22b) + $\frac{C'}{C}\times$ eq. (4.22a) and the use of the bulk equation (4.15) one derives similarly

$$\left[\widehat{L} - \widehat{h}_4\right]_i = 0\,. \tag{4.24}$$

From $(N-1)\times$ eq. (4.22b) - $3\times$ eq. (4.22a) one finds

$$\left[\widehat{h}'_S - (N-1)\frac{C'}{C}\widehat{h}_{yy}\right]_i = 0\,, \tag{4.25}$$

and from the use of (4.25) in (4.22c)

$$\left[\widehat{L}' - \widehat{h}'_4 + 3\frac{A'}{A}\widehat{h}_{yy}\right]_i = 0\,. \tag{4.26}$$

Finally, from eqs. (4.23)–(4.26) in eqs. (4.17a) and (4.18) one derives easily

$$[H_1]_i = [H'_1]_i = 0\,. \tag{4.27}$$

The second order bulk equation (4.21) for the only independent gauge invariant fluctuation H_1, subject to the boundary conditions (4.27) at both branes, has only the trivial solution $H_1 = 0$, and then $H_2 = H_3 = 0$ follows from eqs. (4.19) and (4.20).

Hence there are no physical (gauge invariant) fluctuations at all, notably none with a negative mass^2, that would indicate an instability of the present configuration. Furthermore a similar analysis to the above, for $\widehat{h}_4$, $\widehat{h}_{yy}$ and $\widehat{h}_S$ shows that there are no gauge independent zero modes which simply confirms the fact that there are no parameters left free once the junction conditions are imposed.

A possible hint for the absence of physical fluctuations in the bulk stems from the situation in 5d brane worlds [2, 3], which suffer from a massless radion, but which has no Kaluza-Klein tower [4, 5]. Here the junction conditions eliminate the massless modes in the bulk, hence it is plausible that there are no massive Kaluza-Klein modes left at all.

5. The effect of induced Einstein terms

The presence of induced stress-energy tensors on the branes is motivated, amongst others, as an unavoidable quantum effect of matter on the branes [14]. The same argument can be put forward in favour of the presence of induced Einstein terms on the branes [18]. It is interesting to study how the present setup would react to the presence of these contributions to the actions on the branes, which is what we will present in this section.

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First, the action (2.2) gets now replaced by

$$S = \int d^4x\, d^{N-1}z dy \Big\{ \sqrt{-g^{(D)}}\, \frac{1}{2\kappa_D}\, R^{(D)} + \\ + \sum_{i=1,2} \sqrt{-g^{(D-1)}}\, \delta(y-y_i) \left[\Lambda_i + \frac{1}{2\kappa_i} R^{(D-1)} + \Gamma_i^{(\mathrm{ind})} \right] \Big\}, \qquad (5.1)$$

where the Einstein terms on the branes get multiplied with a priori arbitrary (unknown) coefficients κ_i^{-1}. The only effect of these terms are modifications of the junction conditions of the gravitational field, i.e. of the warp factors A, B, and C. Instead of (2.7) the junction conditions read now [24]

$$\frac{1}{\kappa_D B_i} \left(-3 \left[\frac{A''}{A} \right]_i - (N-1) \left[\frac{C''}{C} \right]_i \right) = -\frac{(N-1)(N-2)}{2\kappa_i C_i^2} + \Lambda_i + \Lambda_i^{(\mathrm{ind},4)}\,, \qquad (5.2a)$$

$$\frac{1}{\kappa_D B_i} \left(-4 \left[\frac{A''}{A} \right]_i - (N-2) \left[\frac{C''}{C} \right]_i \right) = -\frac{(N-3)(N-2)}{2\kappa_i C_i^2} + \Lambda_i + \Lambda_i^{(\mathrm{ind},S)} \qquad (5.2b)$$

leading to

$$\sqrt{f_i}(1-w) + \frac{1}{\sqrt{f_i}}(1+w) = \sigma_i \kappa_D \left(\frac{N-2}{\kappa_i C_i} + C_i \left(\Lambda_i^{(\mathrm{ind},S)} - \Lambda_i^{(\mathrm{ind},4)} \right) \right), \qquad (5.3)$$

instead of eq. (2.9a), whereas eq. (2.9b) remains unchanged.

Note that the counting of free parameters in the configuration of the gravitational field versus the number of equations to be satisfied remains unchanged, hence the need to perform one fine tuning among the parameters in the action remains valid. But, the presence of the induced Einstein terms on the branes will affect the parameters α, r_0 and $|\widehat{y}|$. An analytic solution of eqs. (5.3) and (2.9b) for α, r_0 and $|\widehat{y}|$ is unfortunately impossible, but it is instructive to consider the case where the induced stress-energy tensors Λ_i^{ind} are switched off, and where the coefficients κ_i^{-1} are relatively small:

Once the dimensionless ratio ε_i satisfies

$$\varepsilon_i \equiv \kappa_D^2 \left| \frac{\Lambda_i}{\kappa_i} \right| \ll 1 \qquad (5.4)$$

on each brane, one finds

$$\frac{|\widehat{y}|}{r_0} \sim \max(\varepsilon_i)\,, \qquad (5.5)$$

i.e. one is back in the "close brane limit" as in the case of small induced stress-energy tensors. More precisely, the expressions (2.20a) and (2.20b) for α and r_0, respectively, remain valid, whereas for the distance $\widehat{y}$ between the branes one obtains

$$|\widehat{y}| = -\frac{\kappa_D}{2} \left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2} \right) \left(\frac{w-1}{w+1} \right)^{n_C - \frac{1}{2}}\,. \qquad (5.6)$$

Of utmost importance is the minus sign on the right-hand side of eq. (5.6), which implies that

$$\frac{1}{\kappa_1} + \frac{1}{\kappa_2} < 0 \qquad (5.7)$$

(in the present case with vanishing induced stress-energy tensors $\Lambda_i^{\rm ind}$). If eq. (5.7) is not satisfied, the junction conditions cannot be satisfied by the metric in the bulk, which was assumed to be x^μ-independent. Hence we would expect that in this case either the gravitational field becomes time-dependent, or the branes have to be (anti-)de-Sitter branes, or both.

If we insist on a time independent metric and on Poincare-branes, at least one of the coefficients κ_i^{-1} of the Einstein terms has to have the "wrong" sign. A priori this is not a disaster; what counts, is the absence of ghosts (and tachyons) in the spectrum of gravitational fluctuations. Here, however, we will encounter problems:

Now, in the presence of Einstein terms on the branes, the effective 4d action for 4d gravity gets modified. It is still of the Einstein form (3.10), but the expression (3.11) for the effective 4d (inverse) Planck scale changes. Now it is of the form

$$\frac{1}{\kappa_4} = \Omega^{N-1}\left\{\frac{2}{\kappa_D}\int_0^{\widehat{y}} dy\, A^2(y)\, B(y)\, C(y)^{N-1} + \sum_{i=1,2}\frac{A_i^2 C_i^{N-1}}{\kappa_i}\right\} \tag{5.8}$$

with two additional terms due to the Einstein terms on the branes. In the close brane limit all terms on the right hand side of eq. (5.8) can be computed explicitely, using eqs. (2.20) for α, r_0 and (5.6) for $\widehat{y}$. The net result is that once eq. (5.7) is satisfied, eq. (5.8) gives always a negative gravitational coupling, i.e. a graviton ghost, since the negative contribution to the second term on the right hand side is never completely cancelled by the positive integral dy across the bulk. Consequently Einstein terms on the branes only, without induced stress-energy tensors on the branes, do not allow for a consistent configuration. Only in the presence of induced stress-energy tensors, Einstein terms on the branes would be allowed, provided that their coefficients κ_i^{-1} are not too large.

We have also checked that the absence of gauge invariant scalar fluctuations in the bulk, as discussed in section 4, remains valid in the presence of such Einstein terms on the branes.

6. Conclusions and outlook

The essential result of this paper is a new stabilization mechanism both for the radion and the overall modulus (the size of the compactified extra dimensions parallel to the branes) in higher dimensional brane worlds. It is based on the additional components of the junction conditions, that arise once extra dimensions parallel to the branes are compactified, and notably on the presence of induced terms on the branes.

We have studied this mechanism in detail in the simplest possible setup: higher dimensional brane worlds with an empty bulk, $N-1$ extra dimensions ($N>2$) parallel to the branes compactified on $S^{(N-1)}$, hence a resulting geometry of the form $R^{(3,1)} \times S^{(N-1)} \times S^1/Z_2$.

Matter fields play only an indirect role, namely through the terms on the branes (that affect the junction conditions) induced by their quantum fluctuations. Among the possible induced terms on the branes we have confined ourselves to constant (diagonal) stress-energy tensors and Einstein scalars. We have seen that Einstein scalars cannot play a dominant role; if their coefficients (the sum of which must be negative, cf. eq. (2.19)) are too large, they turn the graviton into a ghost. The induced stress-energy tensors resemble to induced cosmological constants on the branes with the important difference, however, that the diagonal elements along $R^{(3,1)}$ and $S^{(N-1)}$ are generically different. Geometrically this goes hand in hand with the fact that each subspace is of different curvature. This difference turned out to be crucial for the configuration to be consistent, i.e. to satisfy the junction conditions for a real non-singular time-independent metric.

We have attributed the dynamical origin of the difference among the diagonal elements of the stress-energy on the branes to matter-induced radiative corrections as studied in ref. [14]. Its precise dependence on the matter spectrum on the branes and the compactification radius needs eventually additional investigations. Such a difference among the diagonal elements of the stress-energy on the branes could actually be generated at the classical level as well, if form fields with the appropriate rank would live on the branes.

An additional necessary condition for the stability of the configuration is the positivity of all masses^2 of all fluctuations associated to deformations of the configuration. Our somewhat surprising result is that in the present setup there are no such fluctuations (maintaining the shape of $S^{(N-1)}$, but corresponding to variations of the warp factors across the bulk) at all. All scalar fluctuations are frozen.

One should remark that the coupled Einstein field equations in the bulk and the junction conditions on the branes are effects of self-gravity "sourced" by the stress-energy tensors on the branes, and that it is thus pure Einstein gravity that yields the rigidity of the whole setup (at least for scalar fluctuations).

At present we have assumed an empty bulk. The effects of additional fields in the bulk, and generalizations of the compact manifold M^{N-1} have to be studied; only then the present mechanism can eventually be applied to brane worlds motivated by string/M theory.

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A. Field equations and junction conditions for the scalar fluctuations

In this appendix we give the components $R_{\mu\nu}$, R_{yy}, $\gamma^{\alpha\beta}R_{\alpha\beta}$ and $R_{\mu y}$ of the Ricci tensor linear in the fluctuations $\widehat{h}_{\mu\nu}$, $\widehat{h}_{yy}$ and $\widehat{h}_S$ as described in eqs. (3.4)–(3.6). For simplicity we consider only zero modes on $S^{(N-1)}$. Subsequently we give the junction conditions for the fluctuations as derived from eqs. (2.3) linear in the fluctuations.

We use the following notations: the 4d laplacian is denoted by $\Box^{(4)}$, with

$$\Box^{(4)} = g^{\mu\nu}\partial_\mu\partial_\nu = A^{-2}(y)\eta^{\mu\nu}\partial_\mu\partial_\nu\,. \tag{A.1}$$

Derivatives with respect to y are denoted by primes. D' and E' denote the following combinations of y derivatives of the warp functions $A(y)$, $B(y)$ and $C(y)$ (as given in eqs. (2.6)):

$$D' = 4\frac{A'}{A} + (N-1)\frac{C'}{C} - \frac{B'}{B}\,, \qquad E' = 3\frac{A'}{A} + (N-1)\frac{C'}{C}\,. \tag{A.2}$$

The 4d trace of $\widehat{h}_{\mu\nu}$ is denoted by $\widehat{h}_4$:

$$\widehat{h}_4 = \eta^{\mu\nu}\widehat{h}_{\mu\nu}\,(= g^{\mu\nu}h_{\mu\nu})\,. \tag{A.3}$$

Finally, K_μ is defined as (with $\partial^\nu = A^{-2}\eta^{\nu\mu}\partial_\mu$)

$$K_\mu = \partial^\nu\widehat{h}_{\mu\nu} - \frac{1}{2A^2}\partial_\mu\left(\widehat{h}_4 + \widehat{h}_S + \widehat{h}_{yy}\right). \tag{A.4}$$

Then we have

$$\begin{aligned}
A^{-2}R_{\mu\nu} &= \frac{1}{2}\square^{(4)}\widehat{h}_{\mu\nu} - \frac{1}{2}\left(\partial_\mu K_\nu + \partial_\nu K_\mu\right) + \\
&+ \frac{1}{2B^2}\left[\widehat{h}''_{\mu\nu} + \widehat{h}'_{\mu\nu}D' + \eta_{\mu\nu}\frac{A'}{A}\left(\widehat{h}'_4 + \widehat{h}'_S - \widehat{h}'_{yy}\right)\right], && \text{(A.5a)}\\
R_{yy} &= B^2\square^{(4)}\widehat{h}_{yy} - \widehat{h}'_{yy}\left(D' + \frac{B'}{B}\right) + \widehat{h}''_4 + \widehat{h}'_4\left(2\frac{A'}{A} - \frac{B'}{B}\right) + \widehat{h}''_S + \\
&+ \widehat{h}'_S\left(2\frac{C'}{C} - \frac{B'}{B}\right), && \text{(A.5b)}\\
C^{-2}\gamma^{\alpha\beta}R_{\alpha\beta} &= \frac{1}{2}\square^{(4)}\widehat{h}_S + \frac{N-2}{C^2}\widehat{h}_S - \frac{(N-1)(N-2)}{C^2}\widehat{h}_{yy} + \\
&+ \frac{1}{2B^2}\left[\widehat{h}''_S + \widehat{h}'_S D' + (N-1)\frac{C'}{C}\left(\widehat{h}'_4 + \widehat{h}'_S - \widehat{h}'_{yy}\right)\right], && \text{(A.5c)}\\
2R_{\mu y} &= -A^2\partial^\nu\widehat{h}'_{\mu\nu} + \partial_\mu\left(\widehat{h}'_4 + \widehat{h}'_S\right) + \left(\frac{C'}{C} - \frac{A'}{A}\right)\partial_\mu\widehat{h}_S - E'\partial_\mu\widehat{h}_{yy}\,. && \text{(A.5d)}
\end{aligned}$$

Actually we only need the components of the Ricci tensor contracted over μ and ν. Then, instead of the vector K_μ in (A.4), only the combination

$$L = A^2\partial^\mu\partial^\nu\widehat{h}_{\mu\nu} \tag{A.6}$$

appears:

$$\begin{aligned}
g^{\mu\nu}R_{\mu\nu} &= A^{-2}\eta^{\mu\nu}R_{\mu\nu} = \square^{(4)}\widehat{h}_4 - L + \frac{1}{2}\square^{(4)}\left(\widehat{h}_S + \widehat{h}_{yy}\right) + \\
&+ \frac{1}{2B^2}\left[\widehat{h}''_4 + \widehat{h}'_4\left(D' + 4\frac{A'}{A}\right) + 4\frac{A'}{A}\left(\widehat{h}'_S - \widehat{h}'_{yy}\right)\right], && \text{(A.7)}\\
\partial^\mu\partial^\nu R_{\mu\nu} &= \frac{1}{2}\square^{(4)}\left[\square^{(4)}\widehat{h}_4 - L + \square^{(4)}\left(\widehat{h}_S + \widehat{h}_{yy}\right)\right] + \\
&+ \frac{1}{2B^2}\left[L'' + L'\left(D' + 4\frac{A'}{A}\right) + 4\frac{A'^2}{A}L + \frac{A'}{A}\square^{(4)}\left(\widehat{h}'_4 + \widehat{h}'_S - \widehat{h}'_{yy}\right)\right], && \text{(A.8)}\\
2\partial^\mu R_{\mu y} &= -L' - 2\frac{A'}{A}L + \square^{(4)}\left(\widehat{h}'_4 + \widehat{h}'_S\right) + \left(\frac{C'}{C} - \frac{A'}{A}\right)\square^{(4)}\widehat{h}_S - \\
&- \left(3\frac{A'}{A} + (N-1)\frac{C'}{C}\right)\square^{(4)}\widehat{h}_{yy}\,. && \text{(A.9)}
\end{aligned}$$

Next we turn to the junction conditions for the fluctuations. For completeness we include here the Einstein terms on the branes. Then the Einstein equations, including the brane terms from the action (5.1), read (instead of (2.3))

$$\frac{1}{\kappa_D}G^{(D)}_{\mu\nu}=\sum_{i=1,2}\sqrt{\frac{g^{(D-1)}}{g^{(D)}}}\delta(y-y_i)\left[g_{\mu\nu}\Lambda_i-\frac{1}{\kappa_i}G^{(D-1)}_{\mu\nu}+g_{\mu\nu}\Lambda_i^{(\mathrm{ind},4)}\right], \tag{A.10a}$$

$$\frac{1}{\kappa_D}G^{(D)}_{\alpha\beta}=\sum_{i=1,2}\sqrt{\frac{g^{(D-1)}}{g^{(D)}}}\delta(y-y_i)\left[g_{\alpha\beta}\Lambda_i-\frac{1}{\kappa_i}G^{(D-1)}_{\alpha\beta}+g_{\alpha\beta}\Lambda_i^{(\mathrm{ind},S)}\right]. \tag{A.10b}$$

The induced Einstein tensor $G^{(D-1)}_{AB}$ ($A\neq y$ and $B\neq y$) is constructed from g_{AB} with $A\neq y$ and $B\neq y$. Now also the right hand sides of eqs. (A.10) have to be expanded to linear order in the fluctuations. (As stated in section 4, brane bending modes $\beta_i(x^\mu)$ are omitted using a corresponding gauge.) There are terms on the left-hand side of eq. (A.10), to linear order in the fluctuations, which are proportional to the jumps in the y derivatives of the background metric. Nearly all of these terms cancel on both sides once the junction conditions (5.2) for the background metric are used; only terms proportional to $\widehat{h}_{yy}$ are left on the left-hand side.

Again it is useful to use the quantities D', E' defined in eq. (A.2). Instead of the vector K_μ in eq. (A.4), however, it is more useful to introduce $\widehat{K}_\mu$ without $\widehat{h}_{yy}$ (which does not enter the right-hand side of the junction conditions):

$$\widehat{K}_\mu=\partial^\nu\widehat{h}_{\nu\mu}-\frac{1}{2A^2}\partial_\mu\left(\widehat{h}_4+\widehat{h}_S\right). \tag{A.11}$$

After using the junction conditions (5.2a) for the background metric, as stated above, the (μ,ν) components of the junction conditions for the fluctuations read after multiplication with B_i/A_i^2 (where the index i denotes the argument $y=y_i$):

$$\begin{aligned}&\frac{\sigma_i}{B_i\kappa_D}\left[\widehat{h}'_{\mu\nu}+\eta_{\mu\nu}\left(E'\widehat{h}_{yy}-\widehat{h}'_4-\widehat{h}'_S\right)\right]_i= \\ &=\frac{1}{2\kappa_i}\left[-\square^{(4)}\widehat{h}_{\mu\nu}+\left(\partial_\mu\widehat{K}_\nu+\partial_\nu\widehat{K}_\mu\right)+\eta_{\mu\nu}\left(\square^{(4)}\widehat{h}_4+\square^{(4)}\widehat{h}_S-L+\frac{N-2}{C^2}\widehat{h}_S\right)\right]_i\end{aligned} \tag{A.12}$$

where $\sigma_1=+1$, $\sigma_2=-1$.

Since we are only interested in the mode $\widehat{h}_S=\gamma^{\alpha\beta}\widehat{h}_{\alpha\beta}$ with indices in $S^{(N-1)}$, it is sufficient to give the (α,β) components contracted with $\gamma^{\alpha\beta}$. After using the junction conditions (5.2b), and after multiplication with B_i/C_i^2, one obtains

$$\begin{aligned}&\frac{\sigma_i}{B_i\kappa_D}\left[-(N-1)\widehat{h}'_4-(N-2)\widehat{h}'_S+(N-1)\widehat{h}_{yy}\left(4\frac{A'}{A}+(N-2)\frac{C'}{C}\right)\right]_i= \\ &=\frac{1}{2\kappa_i}\left[(N-1)\left(\square^{(4)}\widehat{h}_4-L\right)+(N-2)\square^{(4)}\widehat{h}_S+\frac{(N-2)(N-3)}{2C^2}\widehat{h}_S\right]_i.\end{aligned} \tag{A.13}$$

It is also useful to have the expressions for contracted (μ,ν) components of the junction conditions (A.12). For the contraction of (A.12) with $\eta^{\mu\nu}$ one finds

$$\sigma_i\left[-3\widehat{h}'_4-4\widehat{h}'_S+4E'\widehat{h}_{yy}\right]_i=\frac{B_i\kappa_D}{\kappa_i}\left[\square^{(4)}\widehat{h}_4-L+\frac{3}{2}\square^{(4)}\widehat{h}_S+\frac{2(N-2)}{C^2}\widehat{h}_S\right]_i, \tag{A.14}$$

and from its contraction with derivatives $A^2\partial^\mu\partial^\nu$,

$$\sigma_i \left[L' + 2\frac{A'}{A}L + \Box^{(4)} \left(E'\widehat{h}_{yy} - \widehat{h}'_4 - \widehat{h}'_S \right) \right] = \frac{B_i\kappa_D}{\kappa_i} \left[\frac{(N-2)}{2C^2}\Box^{(4)}\widehat{h}_S \right]_i . \tag{A.15}$$

In the case of massive modes as discussed in section 4 it is useful to remember that, on plane waves of the form (4.4), the action of $\Box^{(4)}$ becomes

$$\Box^{(4)} = -A^{-2}(y)\eta^{\mu\nu}k_\mu k_\nu = A^{-2}(y)\ m^2 . \tag{A.16}$$

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